BASIC MATHEMATICS LOGARITHM

EXERCISE – I

HINTS & SOLUTIONS

Sol.1 **C**

A & B are two rational number then $\frac{A}{B}$ is

Also rational number if $B \neq 0$.

Sol.2 **A**

Every irrational number can be expressed on the number line. This statement is always true.

Sol.3 **D**

$\underset{x}{(\text{rational})} \times \underset{y}{(\text{irrational})}$ = irrational except when $x = 0$

Sol.4 **B**

$(x-2y)\sqrt{2} = (x-2y) + (x-y-1)\sqrt{6}$

$\Rightarrow (x-2y)(\sqrt{2}-1) = (x-y-1)\sqrt{6}$

$\Rightarrow x-2y = 0$ & $x-y-1 = 0 \Rightarrow y = 1, x = 2$

Aliter : $(x+y) + (x-2y)\sqrt{2} = 2x - y + (x-y-1)\sqrt{6}$

$\Rightarrow x(1+\sqrt{2}) + y(1-2\sqrt{2}) = x(2+\sqrt{6})$

$+ y(-1-\sqrt{6}) - \sqrt{6}$ (x, y are rationals)

$\Rightarrow y = \dfrac{(x-1)\sqrt{6} - x\sqrt{2} + x}{\sqrt{6} - 2\sqrt{2} + 2}$

If $x = 2$, $y = 1$ otherwise y is not rational

so $x = 2 \quad y = 1$

Sol.5 **A**

$(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$

$x = 1$ & $x = 2$ & $x = 3 \Rightarrow$ No real solution

Sol.6 **B**

$a(a-b) + b(b-c) + c(c-a) = 0$

$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$

$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0 \Rightarrow a = b = c$

Sol.7 **D**

$$\frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$$

$[\because a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc]$

$\because 3(a-b)(b-c)(c-a) = (a-b)^3 + (b-c)^3 + (c-a)^3$

$\therefore \quad 3 = \dfrac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$

Sol.8 **C**

$x^3 - a^2x + x + 2$ & factor $(x-a)$

then $a^3 - a^2.a + a + 2 = 0$

$\Rightarrow a^3 - a^3 + a + 2 = 0 \Rightarrow a = -2$

Sol.9 **B**

$P(4) = k4^3 + 3 \cdot 4^2 - 3$ & $Q(4) = 2 \cdot 4^3 - 5 \cdot 4 + k$

remainder is same

$P(4) = Q(4) \Rightarrow 64k + 48 - 3 = 128 - 20 + k$

$\Rightarrow 63k = 108 - 45 \quad \Rightarrow k = \frac{63}{63} = 1$

Sol.10 **A**

$2x^3 - 5x^2 + x + 2 = (x-2)(ax^2 - bx - 1)$

$\Rightarrow 2x^3 - 5x^2 + x + 2 = (x-2)(2x^2 - x - 1)$

$a = 2 \Rightarrow b = 1$

Sol.11 **C**

$|4x+3| + |3x-4| = 12$

$-(4x+3) - (3x-4) = 12$; $x \le \frac{-3}{4}$ ...(i)

$4x + 3 - (3x-4) = 12$; $\frac{-3}{4} < x \le \frac{4}{3}$ ...(ii)

$4x + 3 + (3x-4) = 12$; $x > \frac{4}{3}$ ...(iii)

From (i) $x = -\frac{11}{7}$ From (ii) $x = 5$ (reject)

From (iii) $x = \frac{13}{7}$

Sol.12 **D**

$|x|^2 - 3|x| + 2 = 0 \Rightarrow (|x|-2)(|x|-1) = 0$

$\Rightarrow x = \pm 1, \pm 2 \quad \therefore$ number of real roots is 4.

Sol.13 **B**

$= \log_{abc}\sqrt{bc} + \log_{abc}\sqrt{ca} + \log_{abc}\sqrt{ab}$

$= \log_{abc}\sqrt{bc} \cdot \sqrt{ca} \cdot \sqrt{ab} = \log_{abc} abc = 1$

Sol.14 C

$\log_2 15 \log_{1/6} 2 \log_3 \frac{1}{6} = \log_3 15 = 1 + \log_3 5 = 1+1=2$

integral part of $\log_3 5 = 1$

$\therefore$ Integer = 2

Sol.15 D

$\log_x \log_{18} (\sqrt{2} + \sqrt{8}) = \frac{1}{3}$, 1000 x = ?

$\Rightarrow \log_{18} (\sqrt{2} + 2\sqrt{2}) = x^{1/3}$

$\Rightarrow \log_{18} 3\sqrt{2} = x^{1/3}$

$\Rightarrow (3\sqrt{2})^2 = \left(18^{x^{1/3}}\right)^2 \Rightarrow 18 = 18^{2x^{1/3}}$

$\Rightarrow x = \frac{1}{8} \Rightarrow 1000x = 125$

Sol.16 D

$$= \frac{2^{\log_2(a^4)} - 3^{\log_3(a^2+1)} - 2a}{7^{\log_7(a^2)} - a - 1} = \frac{a^4 - (a^2+1) - 2a}{a^2 - a - 1}$$

$$= \frac{(a^2)^2 - (a+1)^2}{(a^2 - a - 1)} = a^2 + a + 1$$

Sol.17 D

$$= \frac{1}{\log_b abc} + \frac{1}{\log_c abc} + \frac{1}{\log_a abc}$$

$= \log_{abc} b + \log_{abc} c + \log_{abc} a = \log_{abc} abc = 1$

Sol.18 B

$\Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x-3)(x+1) = 0$

$\Rightarrow x = -1, 3$ $\quad$ ($x = -1$ reject $\because x > 0$)

number of values of x is one

Sol.19 C

$\sqrt{x^2} = |x| \quad \because x < 0 \Rightarrow |x| = -x$

$\Rightarrow \sqrt{\log_{10}(-x)} = \log_{10}\sqrt{(-x)^2}$

$\Rightarrow \sqrt{\log_{10}(-x)} = \log_{10}(-x)$

$\Rightarrow \log_{10}(-x) = (\log_{10}(-x))^2$

$\Rightarrow \log_{10}(-x)(\log_{10}(-x) - 1) = 0$

$\Rightarrow \log_{10}(-x) = 0$ or $\log_{10}(-x) = 1$

$\therefore (-x) = 1$ or $(-x) = 10$

number of real solution is 2

Sol.20 B

$(3x^2 - 10x + 3) \log |x-3| = 0$

$\Rightarrow \log |x-3| = 0$ or $3x^2 - 10x + 3 = 0$

$\Rightarrow x - 3 \neq 0$ or $(x-3)(3x-1) = 0$

$\Rightarrow x \neq 3$ or $x - 3 = 0, \quad x = 1/3$

and $|x-3| = 1$ But $x \neq 3$

$\Rightarrow x - 3 = 1$ or $3 - x = 1 \Rightarrow x = 4$ or $x = 2$

Three real solution

Sol.21 C

$\log_2 7 \Rightarrow \log_2 4 < \log_2 7 < \log_2 8$

$\Rightarrow 2 < \log_2 7 < 3$ i.e. not integer

Let $\log_2 7 = \frac{p}{q}$ (where p and q are coprime)

$\Rightarrow 2^{p/q} = 7 \Rightarrow 2^p = 7^q$

which is not possible so $\log_2 7$ an irratoinal number

Sol.22 C

$\Rightarrow \text{antilog}_{16} 0.75 = (16)^{3/4} = (2^4)^{3/4} = 2^3 = 8$

EXERCISE – II HINTS & SOLUTIONS

Sol.1 A,B

$\frac{y}{x} = x \Rightarrow x \neq 0 ; y = x^2 \Rightarrow y \neq 0$

$x^2 > 0$ so $y > 0 \Rightarrow y \neq -1 \quad \therefore y \neq -1 \ \& \ 0$

Sol.2 A,B,C,D

$= \log_3 135 \log_3 15 - \log_3 5 \log_3 405$

$= (\log_3 5 + \log_3 3^3)(\log_3 5 + \log_3 3)$

$\quad - \log_3{}^5 (\log_3{}^5 + \log_3 3^4)$

$= (x + 3)(x + 1) - x(x + 4) \quad \{\text{Let } \log_3 5 = x\}$

$= x^2 + 4x + 3 - x^2 - 4x = 3$ Prime, rational Integer

Sol.3 A,B

$\Rightarrow \log_3 xy = 2\log_3 3 + \log_3 2$

$\Rightarrow \log_3 xy = \log_3(2 \times 9) \Rightarrow xy = 18$

and $\log_{27}(x + y) = \frac{2}{3} \Rightarrow x + y = 27^{2/3}$

$\Rightarrow x + y = 3^2 \quad \Rightarrow x + y = 9$

$\therefore \alpha^2 - 9\alpha + 18 = 0 \Rightarrow (\alpha - 6)(\alpha - 3) = 0$

$\Rightarrow \alpha = 6, 3$ so $(x, y) \equiv (6, 3) \equiv (3, 6)$

Sol.4 A,B,C,D

$\Rightarrow \frac{4\log_x 2}{2} + \frac{6\log_x 2}{1 + \log_x 2} = 3$

$\Rightarrow 2(\log_x 2) + 2(\log_x 2)^2 + 6(\log_x 2) = 3 + 3\log_x 2$

$\Rightarrow 2\alpha^2 + 5\alpha - 3 = 0 \quad (\text{Let } \alpha = \log_x 2)$

$\Rightarrow (\alpha + 3)(2\alpha - 1) = 0 \Rightarrow a = -3, 1/2$

$\therefore \log_x 2 = -3 \Rightarrow x = 2^{-1/3}$ (Irrational)

or $\log_x 2 = \frac{1}{2} \Rightarrow x = 4$ (Integer)

Sol.5 A,B,C,D

$[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5](\log_3 x) = \frac{3}{2}$ Let $\log_3 x = \alpha$

$\Rightarrow \frac{(2\alpha^3 - 9\alpha^2 + 10\alpha)}{2} = \frac{3}{2}$

$\Rightarrow 2\alpha^3 - 9\alpha^2 + 10\alpha - 3 = 0$

$\Rightarrow (\alpha - 1)(2\alpha^2 - \alpha + 3) = 0$

$\Rightarrow (\alpha - 1)(\alpha - 3)(2\alpha - 1) = 0 \Rightarrow \alpha = 1, 3, \frac{1}{2}$

$\therefore \log_3 x = 1; \log_3 x = 3 ; \log_3 x = \frac{1}{2}$

$\Rightarrow x = 3 ; x = 3^3 = 27 ; x = \sqrt{3}$

Exactly three solution, one is irrational solution and every real number is also complex.

EXERCISE – III HINTS & SOLUTIONS

Sol.1 Let any two odd natural no.

$(2m+1) \& (2n+1), m, n \in N$

$(2m+1)^2 - (2n+1)^2 = 4m^2 + 4m - 4n^2 - 4n$

$= 4(m - n)(m + n - 1)$

$= 4(m - n)(m - n + (2n + 1))$

Which is divisible by 8.

$\because (m - n)$ is even $\Rightarrow (m + n - 1)$ is odd

$(m - n)$ is odd $\Rightarrow (m + n - 1)$ is even

Sol.2 (i) $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} \times \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}+1}}$

$= \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2} - 1$

(ii) $\frac{1}{1+(\sqrt{2}+\sqrt{3})} \times \frac{1-(\sqrt{2}+\sqrt{3})}{1-(\sqrt{2}+\sqrt{3})}$

$= \frac{1-\sqrt{2}-\sqrt{3}}{1-(\sqrt{2}+\sqrt{3})^2} = \frac{1-\sqrt{2}-\sqrt{3}}{1-(5+2\sqrt{6})} = \frac{1-\sqrt{2}-\sqrt{3}}{-4-2\sqrt{6}}$

$= \frac{-(1-\sqrt{2}-\sqrt{3})}{4+2\sqrt{6}} = \frac{(\sqrt{2}+\sqrt{3}-1)(2-\sqrt{6})}{2(2+\sqrt{6})(2-\sqrt{6})}$

$= \frac{2\sqrt{2}+2\sqrt{3}-2-2\sqrt{3}-3\sqrt{2}+\sqrt{6}}{2(4-6)}$

$= \frac{-\sqrt{2}+\sqrt{6}-2}{2(-2)} = \frac{2+\sqrt{2}-\sqrt{6}}{4}$

Sol.3 **(i)** $(x - y)^3 - y^3$

$= (x - y - y)\ ((x - y)^2 + (x - y)\ y + y^2)$

$= (x - 2y)\ (x^2 - xy + y^2)$

(ii) $a^3 - \dfrac{1}{a^3} + 4$

$= (a)^3 + \left(\dfrac{-1}{a}\right)^3 + (1)^3 - 3(a)\left(\dfrac{-1}{a}\right)(1)$

$= \left(a - \dfrac{1}{a} + 1\right)\left(a^2 + \dfrac{1}{a^2} + 1^2 + 1 + \dfrac{1}{a} - a\right)$

$= \left(a - \dfrac{1}{a} + 1\right)\left(a^2 + \dfrac{1}{a^2} - a + \dfrac{1}{a} + 2\right)$

(iii) $x^3 - 6x^2 + 11x - 6$

at $x = 1$, given polynomial is zero

$= x^2\ (x - 1) - 5x\ (x - 1) + 6\ (x - 1)$

$= (x - 1)\ (x^2 - 5x + 6) = (x - 1)\ (x - 2)\ (x - 3)$

(iv) $x^3 - 9x - 10$

at $x = -2$, polynomial is zero

$= x^2\ (x + 2) - 2x\ (x + 2) - 5(x + 2)$

$= (x + 2)\ (x^2 - 2x - 5)$

(v) $a^2\ (b - c) + b^2\ (c - a) + c^2\ (a - b)$

$= a^2\ (b - c) + b^2c - b^2a + c^2a - c^2b$

$= a^2(b - c) + bc\ (b - c) - a(b^2 - c^2)$

$= (b - c)\ [a^2 + bc - ab - ac]$

$= (b - c)\ [a(a - b) - c\ (a - b)]$

$= (b - c)\ (a - b)\ (a - c)$

$= -\ (a - b)\ (b - c)\ (c - a)$

Sol.4 **(i)** $(1 + x^4 + x^8)$

$= (x^4)^2 + 2x^4 + (1)^2 - x^4$

$= (x^4 + 1)^2 - x^4 = (x^4 + 1)^2 - (x^2)^2$

$= (x^4 + x^2 + 1)\ (x^4 - x^2 + 1)$

$= (x^4 - x^2 + 1)\ [(x^2 + 1)^2 - x^2]$

$= (x^4 - x^2 + 1)\ (x^2 - x + 1)\ (x^2 + x + 1)$

(ii) $x^4 + 4$

$= (x^2)^2 + (2)^2 + 4x^2 - 4x^2 = (x^2 + 2)^2 - (2x)^2$

$= (x^2 - 2x + 2)\ (x^2 + 2x + 2)$

Sol.5 Let $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f} = k \Rightarrow a = bk,\ c = dk,\ e = fk$

$\therefore \dfrac{2b^6k^4 + 3b^2d^2k^4 - 5f^5k^4}{2b^6 + 3b^2d^2 - 5f^5}$

$= \dfrac{k^4(2b^6 + 3b^2d^2 - 5f^5)}{(2b^6 + 3b^2d^2 - 6f^5)} = k^4 = \left(\dfrac{a}{b}\right)^4 = \dfrac{a^4}{b^4}$

Sol.6 $|a_1| + |a_2| + |a_3| + \ldots + |a_n| = 0$,

sum of positive number is zero

then all $a_1, a_2 \ldots a_n$ must be zero

$a_1 = a_2 = a_3 = \ldots = a_n = 0$

Sol.7 **(i)** $|x| + 2 = 3 \Rightarrow |x| = 1 \Rightarrow x = \pm 1$

(ii) $|x| - 2x + 5 = 0$

If $x \geq 0 \Rightarrow x - 2x + 5 = 0 \Rightarrow -x + 5 = 0 \Rightarrow x = 5$

If $x < 0 \Rightarrow -x - 2x + 5 = 0 \Rightarrow -3x + 5 = 0$

$\Rightarrow x = \dfrac{5}{3}$ (reject)

It is not satisfies the given equation so $x = 5$

(iii) $x\ |x| = 4$

If $x \geq 0 \Rightarrow x^2 = 4 \Rightarrow x = 2,\ x \neq -2$

If $x < 0 \Rightarrow -x^2 = 4 \Rightarrow x^2 = -4$

(iv) $||x - 1| - 2| = 1$

$\Rightarrow |x - 1| - 2 = 1$ or $|x - 1| - 2 = -1$

$\Rightarrow |x - 1| = 3$ or $|x - 1| = 1$

$\Rightarrow x = 4\ \&\ x = -2$ or $x = 2\ \&\ x = 0$

$\therefore x = 0, \pm 2, 4$

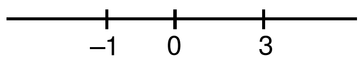
(v) $|x|^2 - |x| + 4 = 2x^2 - 3|x| + 1$

$\Rightarrow 2|x|^2 - |x|^2 - 2|x| - 3 = 0$

$\Rightarrow |x|^2 - 2|x| - 3 = 0 \Rightarrow (|x| - 3)\ (|x| + 1) = 0$

$\Rightarrow |x| = 3\ \&\ |x| + 1 \neq 0 \therefore x = \pm 3\ \&\ |x| \neq -1$

(vi) $|x - 3| + 2|x + 1| = 4$



$-(x - 3) - 2\ (x + 1) = 4;\ x < -1 \Rightarrow x = -1$ (reject)

$-(x - 3) + 2\ (x + 1) = 4\ ;\ -1 \leq x < 3 \Rightarrow x = -1$ (accept)

$x - 3 + 2\ (x + 1) = 4\ ;\ 3 \geq x \Rightarrow x = \dfrac{5}{3} \notin [3, \infty)$ (reject)

So $x = -1$

(vii) $||x - 1| - 2| = |x - 3|$

$|x - 1| - 2 = x - 3$ or $|x - 1| - 2 = -x + 3$

$|x - 1| = x - 1$ or $|x - 1| = -x + 5$

$\begin{cases} x - 1 = x - 1\ ;\ x \geq 1 \\ -x + 1 = x - 1\ ;\ x < 1 \end{cases}$ | $\begin{cases} -x + 1 = -x + 5\ ;\ x < 1 \\ x - 1 = -x + 5\ ;\ x \geq 1 \end{cases}$

$\Rightarrow \begin{cases} \text{identity}\ ;\ x \geq 1 \\ 2x = 2\ ;\ x < 1 \\ x = 1 \end{cases}$ | $\Rightarrow \begin{cases} \text{no solution}\ ;\ x < 1 \\ 2x = 6\ ;\ x < 1 \\ x = 3 \end{cases}$

$x = 1, 3,\ x \geq 1 \Rightarrow x \in [1, \infty)$

Sol.8 $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

$\because 7^{\log_3 5} = 5^{\log_3 7}$

and $3^{\log_5 7} = 7^{\log_5 3}$ putting these values

$5^{\log_3 7} + 7^{\log_5 3} - 5^{\log_3 7} - 7^{\log_5 3} = 0$

Sol.9 $4^{\log_{16} 4} + 9^{\log_3 9} = 10^{\log_x 83}$

$\Rightarrow 4^{\log_{4^2} 4} + 9^{\log_3 3^2} = 10^{\log_x 83}$

$\Rightarrow 4^{\frac{1}{2}\log_4 4} + 9^{2\log_3 3} = 10^{\log_x 83}$

$\Rightarrow 2 + 81 = 10^{\log_x 83} \Rightarrow 83 = 10^{\log_x 83}$

$\Rightarrow \log_{10}83 = \log_x 83 \ \log_{10}10 \ \therefore \ x = 10$

Sol.10 a, b, c are different (+) real no. $\neq 1$

$$\Rightarrow \frac{1}{\log_a b.\log_a c} + \log_a b.\frac{\log_a b}{\log_a c} + \log_a c.\frac{\log_a c}{\log_a b} = 3$$

$$\Rightarrow \frac{(1)^3 + (\log_a b)^3 + (\log_a c)^3}{(1)\log_a b \log_a c} = 3$$

$\Rightarrow (\log_a a)^3 + (\log_a b)^3 + (\log_a c)^3 = 3(\log_a a)(\log_a b)(\log_a c)$

$x^3 + y^3 + z^3 = 3\ xyz \Rightarrow x + y + z = 0$ or $x=y=z$

$\Rightarrow \log_a a + \log_a b + \log_a c = 0$ or $\log_a a \neq \log_a b \neq \log_a c$

$\Rightarrow \log_a abc = 0 \Rightarrow abc = a^0 \Rightarrow abc = 1$

Sol.11 $a = \log_{12}18$ & $b = \log_{24} 54$

$$a = \frac{\log_2 18}{\log_2 12} = \left(\frac{2\log_2 3 + 1}{\log_2 3 + 2}\right),\ b = \frac{\log_2 54}{\log_2 24} = \left(\frac{3\log_2 3 + 1}{\log_2 3 + 3}\right)$$

Let $\log_2 3 = x \Rightarrow a = \left(\frac{2x+1}{x+2}\right)\ b = \left(\frac{3x+1}{x+3}\right)$

$$ab + 5(a-b) = \frac{(2x+1)(3x+1)}{(x+2)(x+3)} + 5\left(\frac{2x+1}{x+2} - \frac{3x+1}{x+3}\right)$$

$$= \frac{6x^2 + 5x + 1 + 5(2x^2 + 7x + 3 - 3x^2 - 7x - 2)}{(x+2)(x+3)}$$

$$= \frac{x^2 + 5x + 6}{(x+2)(x+3)} = \frac{(x+2)(x+3)}{(x+2)(x+3)} = 1$$

Sol.12 $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b} = k$

$a = e^{k(b-c)} \Rightarrow a^a = e^{k(ab - ca)}$

$b = e^{k(c-a)} \Rightarrow b^b = e^{k(bc - ab)}$

$c = e^{k(a-b)} \Rightarrow c^c = e^{k(ac - ab)}$

$\Rightarrow a^a . b^b . c^c - e^{k(ab-ca+bc-ab+ca-ab)}$

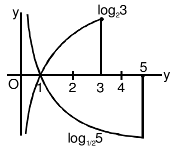
$\Rightarrow a^a . b^b . c^c = e^0 = 1$

Sol.13 **(a)** $x = \log_2 3$

or $y = \log_{1/2} 5$

$\log_2 3 > -\log_2 5$

$\Rightarrow \log_2 3 > \log_{1/2} 5$



(b) $x = \log_7 11$, $y = \log_8 5$

$x = \log_7 11 > 1$, $y = \log_8 5 < 1$

$\therefore\ x > y \Rightarrow \log_7 11 > \log_8 5$

Sol.14 $\log_{10}(x^2 - 12x + 36) = 2$

$\Rightarrow x^2 - 12x + 36 = 100 \Rightarrow x^2 - 12x - 64 = 0$

$\Rightarrow (x-16)(x+4) = 0 \Rightarrow x = 16, -4$

Sol.15 $\log_4\log_3\log_2 x = 0 \Rightarrow \log_3 \log_2 x = 4^0 = 1$

$\Rightarrow \log_2 x = 3^1 \Rightarrow x = 2^3\ \therefore\ x = 8$

Sol.16 $\log_3(\log_9 x + \frac{1}{2} + 9^x) = 2x$

$\Rightarrow \log_9 x + \frac{1}{2} + 9^x = 3^{2x} \Rightarrow \log_9 x + \frac{1}{2} + 9^x = 9^x$

$\Rightarrow \log_9 x = -\frac{1}{2} \Rightarrow x = 9^{-1/2} \Rightarrow x = \frac{1}{\sqrt{9}} \Rightarrow x = \frac{1}{3}$

Sol.17 $2\log_4(4-x) = 4 - \log_2(-2-x)$

here $4 - x > 0 \Rightarrow 4 > x$ & $-2 - x > 0 \Rightarrow -2 > x$

$\Rightarrow \log_2(4-x) + \log_2(-2-x) = 4$

$\Rightarrow (4-x)(2+x) = (-16)$

$\Rightarrow x^2 - 2x - 24 = 0 \Rightarrow (x-6)(x+4) = 0$

$\Rightarrow x = 6, x = -4 \therefore x = -4$

Sol.18 $(\log_{10}x)^2 + 2\log_{10}x + 1 = (\log_{10}2)^2$

$\Rightarrow (\log_{10}x + 1)^2 = (\log_{10}2)^2$

$\Rightarrow \log_{10}(10x) = \pm\log_{10}2$

$\Rightarrow x = 1/5$ or $x = 1/20$

Sol.19 $x^{\frac{\log x+5}{3}} = 10^{3+\log x} \Rightarrow x^{\log x+5} = 10^{3(\log x+5)}$

$\Rightarrow x^{(\log x+5)} = 1000^{(\log x+5)} \Rightarrow x = 1000 = 10^3$ &

If $\log_{10}x + 5 = 0$ then x will satisfy the equation

$x = 10^{-5}$, $x = 10^3, 10^{-5}$

Sol.20 $(\log_5 x)^2 + \log_{5x}\left(\frac{5}{x}\right) = 1$

$$\Rightarrow (\log_5 x)^2 + \frac{\log_5\left(\frac{5}{x}\right)}{1 + \log_5 x} = 1$$

$\Rightarrow (\log_5 x)^2 + \dfrac{\log_5 5 - \log_5 x}{1+\log_5 x} = 1$

Let $\log_5 x = \alpha$

$\Rightarrow a^2 + \dfrac{1-\alpha}{1+\alpha} = 1 \Rightarrow \alpha^3 + \alpha^2 + 1 - \alpha = 1 + \alpha$

$\Rightarrow \alpha(\alpha^2 + \alpha - 2) = 0 \Rightarrow \alpha(\alpha + 2)(\alpha - 1) = 0$

$\therefore \alpha = 0, 1, -2$

$\log_5 x = 0 \Rightarrow x = 5^0 \Rightarrow x = 1$

$\log_5 x = 1 \Rightarrow x = 5^1 \Rightarrow x = 5$

$\log_5 x = -2 \Rightarrow x = 5^{-2} \Rightarrow x = \dfrac{1}{25}$

Sol.21 $\log_4(\log_2 x) + \log_2(\log_4 x) = 2$

$\Rightarrow \log_{2^2}(\log_2 x) + \log_2(\log_4 x) = 2$

$\Rightarrow \dfrac{1}{2}\log_2(\log_2 x) + \log_2(\log_4 x) = 2$

$\Rightarrow \log_2(\log_2 x) + 2\log_2(\log_4 x) = 4$

$\Rightarrow \log_2\{(\log_2 x)(\log_4 x)^2\} = 4$

$\Rightarrow (\log_2 x)\left(\dfrac{\log_2 x}{2}\right)^2 = 2^4$

$\Rightarrow (\log_2 x)^3 = 2^6 = (2^2)^3$

$\Rightarrow \log_2 x = 4 \Rightarrow x = 2^4 \Rightarrow x = 16$

Sol.22 $5^x . \sqrt[x]{8^{x-1}} = 500 \Rightarrow 5^x . 8^{\frac{x-1}{x}} = 5^3 . 2^2$

$\Rightarrow 5^{x-3}\, 2^{\frac{3x-3}{x}-2} = 1 \Rightarrow 5^{(x-3)}\, 2^{\frac{3x-3}{x}-2} = 1$

5 & 2 are coprime no. If their multiply is one. So individual power of 5 & 2 is zero

$x - 3 = 0, \dfrac{x-3}{x} = 0 \Rightarrow x = 3$

Sol.23 (a) Number of integer 6^{15}

Let $x = 6^{15}$

$\log_{10}x = 15\log_{10}6 = 15[\log_{10}2 + \log_{10}3]$

$\approx 15[0.3010 + 0.4771]$

$\approx 15[0.7781]$

≈ 11.6715

Number of integer of $x = 6^{15}$ is 12.

(b) $x = 3^{-100}$

$\log_{10}x = -100\log_{10}3 = -100(0.4471)$

$= -47.71 = -47 - 1 + 1 - 0.71$

$= -48 + 0.29 = \overline{48}.29$

Number of zeros after decimal is 47.

Sol.24 $\log_{100}|x + y| = \dfrac{1}{2}$; $\log_{10}y - \log_{10}|x| = \log_{100}4$

$|x + y| = 10 \Rightarrow x + y = 10 \quad \because x + y > 0$

$\because y > 0 \quad \therefore \log_{10}\dfrac{y}{|x|} = \log_{10^2}4$

$\Rightarrow \dfrac{y}{|x|} = 2 \Rightarrow y = 2|x| \quad \therefore y > 0$

$x - 2x = \pm 10$

$x = \pm\dfrac{10}{3}$ $\qquad -x = \pm 10$

Here $x > 0$ & $y > 0$ $\qquad \therefore y > 0$ so $x < 0$

$x = \dfrac{10}{3}, y = \dfrac{20}{3}$ $\qquad x = -10, y = 20$

Sol.25 $|x - 1|^A = (x - 1)^7$ $\qquad A = \log_3 x^2 - 2\log_x 9$

L.H.S. $> 0 \Rightarrow$ R.H.S. $> 0 \Rightarrow x - 1 > 0$

$[2\log_3 x - 4\log_x 3]\log_3(x - 1) = 7\log_3(x - 1)$

$\Rightarrow [2\log_3 x - 4\log_x 3 - 7]\log_3(x - 1) = 0$

$\therefore$ If $\log_3(x - 1) = 0 \Rightarrow x - 1 = 1 \Rightarrow x = 2$

$2\log_3 x - 4\log_x 3 - 7 = 0$. let $\log_3 x = t$

$2t^2 - 7t - 4 = 0 \Rightarrow (t - 4)(2t + 1) = 0$

$t = 4 \Rightarrow x = 81$ or $t = -1/2 \Rightarrow x = 1/\sqrt{3}$ (Reject)

$x > 1$ so $x = 2, 81$

Sol.26 $2\log_2\log_2 x + \log_{1/2}\log_2(2\sqrt{2}\,x) = 1$

$\Rightarrow \log_2(\log_2 x)^2 + \log_2\left(\dfrac{1}{\log_2 2\sqrt{2}x}\right) = 1$

$\Rightarrow \log_2\left[\dfrac{(\log_2 x)^2}{\log_2 2^{3/2} + \log_2 x}\right] = 1$

$\Rightarrow (\log_2 x)^2 = 2\left[\dfrac{3}{2} + \log_2 x\right]$

$\Rightarrow (\log_2 x)^2 - 2(\log_2 x) - 3 = 0$

$\Rightarrow \log_2 x = 3$ or $\log_2 x = -1$ (not possible)

$x = 2^3 \Rightarrow x = 8 \quad (\because \log_2 x > 1)$

Sol.27 $A = \log_{10}\left(\dfrac{ab+\sqrt{ab^2-4(a+b)}}{2}\right)\left(\dfrac{ab-\sqrt{ab^2-4(a+b)}}{2}\right)$

$A = \log_{10}\left(\dfrac{(ab)^2 - (ab)^2 + 4(a+b)}{4}\right)$

$A = \log_{10}(a+b) = \log_{10}(43+57) = \log_{10}100 = 2$

$A.B = 2\,(2^{\log_6 18}).(3^{\log_6 3}) = 2^{2(1+\log_6 3)}\,.\,3^{\log_6 3}$

$= 4(2.3)^{\log_6 3} = 4.3 = 12$

Sol.28 **(a)** $x = \log_3 4$ & $y = \log_5 3$

$\log_3 10 = \log_3 2 + \log_3 5 = \frac{1}{2}\,2\log_3 2 + \log_3 5$

$= \frac{1}{2}\log_3 4 + \frac{1}{\log_5 3} = \frac{x}{2} + \frac{1}{y} = \frac{xy+2}{2y}$

& $\log_3(1.2) = \log_3 \frac{12}{10} = \log_3 12 - \log_3 10$

$= \log_3 4 + \log_3 3 - \log_3 10 = x + 1 - \frac{x}{2} - \frac{1}{y}$

$= \frac{2xy + 2y - xy - 2}{2y} = \frac{xy + 2y - 2}{2y}$

(b) $k^{\log_2 5} = 16$ then $k^{(\log_2 5)^2} = (k^{\log_2 5})^{\log_2 5}$

$= (16)^{\log_2 5} = 2^{4\log_2 5} = 2^{\log_2 5^4} = 5^4 = 625$

Sol.29 **(a)** $\log_{10}(x^2 - 12x + 36) = 2 \Rightarrow \log_{10}(x-6)^2 = 2$

$\Rightarrow (x-6)^2 = 10^2 \quad \Rightarrow (x-6)^2 = (10)^2$

$\Rightarrow (x-6)^2 = 10 \quad \Rightarrow x = 4, x = 16$

(b) $9^{1+\log_3 x} - 3^{1+\log_3 x} = 210$

$9.9^{\log_3 x} - 3x = 210$

$\Rightarrow 9x^2 - 3x - 210 = 0 \Rightarrow 3x^2 - x - 70 = 0$

$\Rightarrow (x-5)(3x+14) = 0$

$\therefore\ x = 5, x = -\frac{14}{3}$ But $x > 0$

Sol.30 **(a)** $\log_{1/3} \sqrt[4]{729\sqrt[3]{9^{-1}27^{-4/3}}}$

$= \log_{1/3} \sqrt[4]{729\sqrt[3]{3^{-2}3^{-4}}}$

$= \log_{1/3} \sqrt[4]{729.3^{-6/3}} = \log_{1/3} \sqrt[4]{27^2.3^{-2}}$

$= \log_{1/3} \sqrt[4]{9^2} = \log_{1/3} 9^{2/4} = \log_{1/3} 3 = -1$

(b) $a^{\left\{\frac{\log_b(\log_b N)}{\log_b a}\right\}}$

$= a^{(\log_a b)(\log_b(\log_b N))} = b^{\log_b(\log_b N)} = \log_b N$

Sol.31 **(a)** $\log_4\log_3\log_2 x = 0$

$\Rightarrow \log_3\log_2 x = 4^0 = 1 \Rightarrow \log_2 x = 3^1$

$\Rightarrow x = 2^3 \Rightarrow x = 2^3 \Rightarrow x = 8$

(b) $\log_e\log_5[\sqrt{2x-2} + 3] = 0$

$\Rightarrow \log_5[\sqrt{2x-2} + 3] = e^0$

$\Rightarrow \sqrt{2x-2} + 3 = 5 \Rightarrow \sqrt{2x-2} = 2$

$\Rightarrow 2x - 2 = 4 \Rightarrow 2x = 6 \Rightarrow x = 3$

Sol.32 **(a)** $\log_\pi 2 + \log_2\pi$

We know $\log_2\pi > 1$

$\Rightarrow (\log_2\pi - 1)^2 > 0 \Rightarrow (\log_2\pi)^2 + 1 > 2\log_2\pi$

$\Rightarrow \frac{(\log_2\pi)^2 + 1}{\log_2\pi} > 2 \Rightarrow (\log_2\pi + \log_\pi 2) > 2$

(b) $\log_3 5 = \frac{p}{q}$ (where p & q are integer)

$\Rightarrow 5 = 3^{p/q} \Rightarrow 5^q = 3^p$ Which is not possible

($\because$ 5 & 3 are coprime no.)

So $\log_3 5$ is an irrational

Similarly $\log_2 7$ is also irratoinal

Sol.33 $a, b \in R$ greater than one

$\exists\ c \in R^+$ & $c \neq 1$

s.t. $2(\log_a c + \log_b c) = 9\log_{ab} c$

$\Rightarrow 2\left(\log_a c + \frac{\log_a c}{\log_a b}\right) = \frac{9\log_a c}{(1+\log_a b)}$

$\Rightarrow 2\log_a c\left(\frac{1+\log_a b}{\log_a b}\right) = \frac{9\log_a c}{(1+\log_a b)}$

$\Rightarrow 2(1 + \log_a b)^2 = 9\log_a b \quad \{A = \log_a b\}$

$\Rightarrow 2A^2 + 4A + 2 = 9A \Rightarrow 2A^2 - 5A + 2 = 0$

$\Rightarrow 2A^2 - 4A - A + 2 = 0$

$\Rightarrow 2A(A-2) - 1(A-2) = 0 \Rightarrow A = 2$ or $A = \frac{1}{2}$

Largest value of $A = \log_a b$ is 2.

Sol.34 $\log_3 x\,.\,\log_4 x\,.\,\log_5 x = \log_3 x\,.\,\log_4 x + \log_4 x\,.\,\log_5 x + \log_5 x\,\log_3 x$

$\Rightarrow \log_3 x\,\log_4 x\,\log_5 x\left(\frac{1}{\log_3 x} + \frac{1}{\log_4 x} + \frac{1}{\log_5 x} - 1\right) = 0$

$\Rightarrow \log_x 3 + \log_x 4 + \log_x 5 = 1$

$\Rightarrow \log_x(3\,.\,4\,.\,5) = 1 \Rightarrow \log_x 60 = 1 \Rightarrow x = 60$

or $\log_3 x\,\log_4 x\,\log_5 x = 0$

or $\log_3 x = 0$ or $\log_4 x = 0$ or $\log_5 x = 0$

$\Rightarrow x = 1$ or $x = 1$ or $x = 1$

$\therefore$ Square of the sum $= (60 + 1)^2 = 3721$

Sol.35 $\dfrac{2}{\log_4(2000)^6} + \dfrac{3}{\log_5(2000)^6}$

$= \dfrac{1}{3}\log_{2000} 4 + \dfrac{1}{2}\log_{2000} 5$

$= \dfrac{1}{6}\log_{2000}(4^2.5^2) = \dfrac{1}{6}$

Sol.36 $4^{5\log_{4\sqrt{2}}(3-\sqrt{6})-6\log_8(\sqrt{3}-\sqrt{2})}$

$= (2^2)^{5\log_{2^{5/2}}(3-\sqrt{6})-6\log_{2^3}(\sqrt{3}-\sqrt{2})}$

$= 2^{2\times5\times\frac{2}{5}\log_2(3-\sqrt{6})-\frac{2\times6}{3}\log_2(\sqrt{3}-\sqrt{2})}$

$= 2^{\log_2\frac{(3-\sqrt{6})^4}{(\sqrt{3}-\sqrt{2})^4}} = (\sqrt{3})^4\dfrac{(\sqrt{3}-\sqrt{2})^4}{(\sqrt{3}-\sqrt{2})^4} = 9$

Sol.37 $\dfrac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\ [(\sqrt{7})^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6}]$

$= \dfrac{9^{2\log_9 5} + 3^{3\log_3\sqrt{6}}}{409}\ (7^{\log_7 25} - 5^{\frac{3}{2}\log_5 6})$

$= \dfrac{25+6^{3/2}}{409}\ [25 - 6^{3/2}] = \dfrac{25^2 - 6^{\frac{3}{2}\times2}}{409} = \dfrac{409}{409} = 1$

Sol.38 $5^{\log_{1/5}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}}\dfrac{4}{\sqrt{7}+\sqrt{3}} + \log_{1/2}\dfrac{1}{10+2\sqrt{21}}$

$= 5^{\log_5 2} + 2\log_2\left(\dfrac{4}{\sqrt{7}+\sqrt{3}}\right) + \log_2(10+2\sqrt{21})$

$= 2 + \log_2\dfrac{16}{(\sqrt{7}+\sqrt{3})^2} \times (10 + 2\sqrt{21})$

$= 2 + \log_2 16 = 2 + 4\log_2 2 = 2 + 4 = 6$

Sol.39 $4^{\log_{10}x+1} - 6^{\log_{10}x} - 2.3^{\log_{10}x^2+2} = 0$

$\Rightarrow 2^{\log_{10}x^2+2} - (2.3)^{\log_{10}x} - 2.3^2 . 3^{\log_{10}x^2} = 0$

$\Rightarrow 4.2^{2\log_{10}x} - 2^{\log_{10}x} . 3^{\log_{10}x} - 18.3^{2\log_{10}x} = 0$

$\Rightarrow 4a^2 - a.b - 18b^2 = 0$

$\Rightarrow 4a^2 + 8ab - 9ab - 18b^2 = 0$

$\Rightarrow (a + 2b)(4a - 9b) = 0$

$\Rightarrow a + 2b = 0$ or $4a - 9b = 0$

so $4a - 9b = 0 \Rightarrow 4.2^{\log_{10}x} - 9.3^{\log_{10}x} = 0$

$\Rightarrow 2^2 . 2^{\log_{10}x} = 3^2 . 3^{\log_{10}x}$

$\Rightarrow 2^{\log_{10}x+2} = 3^{\log_{10}x+2}$

$\Rightarrow \log_{10}x + 2 = 0 \Rightarrow \log_{10}x = -2$

$\Rightarrow x = 10^2 \Rightarrow x = 10^{-2} \Rightarrow x = 1/100$

Which is not possible because

or $a + 2b = 0 \Rightarrow 2^{\log_{10}x} > 0 \Rightarrow 3^{\log_{10}x} > 0$

Sum of two positive no., is not equal to zero.

So $x = 1/100$

Sol.40 Given that

$\log_2 a^2 = 2S$, $\log_2 b^5 = 5(2S^2)$ & $\log_2 c^4 = 3(S^3 + 1)$

then $\log_2\dfrac{a^2b^2}{c^4} = 2S + 10S^2 - 3(S^3 + 1)$

Sol.41 $49^{(1-\log_7 2)} + 5^{-\log_5 4}$

$= 49^{\log_7\left(\frac{7}{2}\right)} + 5^{\log_5\left(\frac{1}{4}\right)} = 7^{2\log_7\left(\frac{7}{2}\right)} + \dfrac{1}{4}$

$= \left(\dfrac{7}{2}\right)^2 + \dfrac{1}{4} = \dfrac{49}{4} + \dfrac{1}{4} = \dfrac{50}{4} = \dfrac{25}{2}$

Sol.42 $\log_2 3 = a$, $\log_3 5 = b$, $\log_7 2 = c$

$\log_{140}63 = \dfrac{\log_2 63}{\log_2 140} = \dfrac{\log_2(7.3^2)}{\log_2(2^2 5.7)}$

$= \dfrac{\log_2 7 + 2\log_2 3}{2 + \log_2 5 + \log_2 7} = \dfrac{\frac{1}{c} + 2a}{2 + ab + \frac{1}{c}}$

$= \dfrac{1+2ac}{1+abc+2c}$

Sol.43 $x = \sqrt{\log_a b}$ & $y = \sqrt{\log_b a}$

$a^x = a^{\frac{\log_a b}{\sqrt{\log_a b}}} \Rightarrow a^x - b^y = 0$

EXERCISE – IV HINTS & SOLUTIONS

Sol.1 $\log_a N . \log_b N + \log_b N + \log_c N + \log_a N$

$$= \frac{\log_a N.\log_b N\log_c N}{\log_{abc} N}$$

$$\text{L.H.S.} = \frac{1}{\log_N a\log_N b} + \frac{1}{\log_N b\log_N c} + \frac{1}{\log_N c\log_N a}$$

$$= \frac{\log_N c + \log_N a + \log_N b}{\log_N a.\log_N b\log_N c} = \frac{\log_N abc}{\log_N a\log_N \log_N c}$$

$$= \frac{\log_a N\log_b N\log_c N}{\log_{abc} N} = \text{R.H.S.}$$

Sol.2 **(a)** $\frac{\log_{10}(x-3)}{\log_{10}(x^2-21)} = \frac{1}{2}$

$\Rightarrow \log_{10}(x-3)^2 = \log_{10}(x^2-21)$

$\Rightarrow x^2 - 6x + 9 = x^2 - 21 \Rightarrow 6x = 30 \Rightarrow x = 5$

(b) $\log_{10}(\log_{10}x) + \log_{10}(\log_{10}x^3 - 2) = 0$

Here $\log_{10} x > 0$

$\Rightarrow \log_{10}[(\log_{10}x).(\log_{10}x^3 - 2)] = 0$

$\Rightarrow (\log_{10}x)(\log_{10}x^3 - 2) = 1$

$\Rightarrow (\log_{10}x)(3\log_{10}x - 2) = 1$

Let $\log_{10}x = y$

$\Rightarrow y(3y-2) = 1 \quad \Rightarrow 3y^2 - 2y = 1$

$\Rightarrow 3y^2 - 2y - 1 = 0 \quad \Rightarrow (y-1)(3y+1) = 0$

$\log_{10}x = 1 \quad \Rightarrow x = 10$

$\log_{10}x = -\frac{1}{3} \Rightarrow$ Not possible ($\because \log_{10} x > 0$)

(c) $\log_x 2 . \log_{2x} 2 = \log_{4x} 2$

$\Rightarrow \frac{1}{\log_2 x} \cdot \frac{1}{\log_2 2x} = \frac{1}{\log_2 4x}$

$\Rightarrow \log_2 x \log_2 2x = \log_2 4x$

$\Rightarrow \log_2 x(1 + \log_2 x) = (2 + \log_2 x)$

$\Rightarrow y + y^2 = 2 + y \quad$ {let $y = \log_2 x$ }

$\Rightarrow y = \pm\sqrt{2} \Rightarrow \log_2 x = \pm\sqrt{2} \Rightarrow x = 2^{\pm\sqrt{2}}$

(d) $t + 5t = 3 \Rightarrow t = \frac{1}{2},\ x = 2^{-\log_5 a}$

Sol.3 $\log_x y + \log_y x = \frac{10}{3}$ & $xy = 144$, $\frac{x+y}{2} = \sqrt{N}$

$\Rightarrow \alpha + \frac{1}{\alpha} = \frac{10}{3}$ (let $\alpha = \log_x y$)

$\Rightarrow 3\alpha^2 + 3 = 10\alpha \Rightarrow 3\alpha^2 - 10\alpha + 3 = 0$

$\Rightarrow (\alpha - 3)(3\alpha - 1) = 0$

$\therefore \log_x y = 3,\ \log_x y = 1/3 \Rightarrow y = x^3,\ y = x^{1/3}$

$y = x^3, y = \frac{144}{x}$	$x = y^3, x = \frac{144}{y}$
$\Rightarrow x^4 = 144 \Rightarrow x^2 = 12$, (–12 reject)	$x = 24\sqrt{3}$, $y = 2\sqrt{3}$
$\Rightarrow x = 2\sqrt{3}, y = 24\sqrt{3}$	

$$\frac{x+y}{2} = \frac{26\sqrt{3}}{2} = 13\sqrt{3} = \sqrt{169\times 3} = \sqrt{507}$$

N = 507

(d) $5^{\log_a x} + 5x^{\log_a 5} = 3$

$\Rightarrow 5^{\log_a x} + 5.5^{\log_a x} = 3 \Rightarrow 6.5^{\log_a x} = 3$

$\Rightarrow 5^{\log_a x} = \frac{1}{2} \Rightarrow 5^{\log_a x} = 2^{-1} \Rightarrow \log_a x = -\log_5 2$

$\Rightarrow x = a^{-\log_5 2} \quad \Rightarrow x = 2^{-\log_5 a}$

Sol.4 **(a)** Given $\log_{10} 34.56 = 1.5386$

then $\log_{10}(3.456) = 0.5386$

$\log_{10}(0.3456) = \bar{1}.5386$

$\log_{10}(0.003456) = \bar{3}.5386$

(b) Characterstic is 3 i.e. but not equal to 4

$\log_7 N = 3 \Rightarrow N = 7^3$ & $\log_7 N < 4 \Rightarrow N < 7^4$

N is positive integer

Number of N is $= 7^4 - 7^3$	No. between
$= 7^3(7-1)$	7^3 & 7^4
$= 2058$	$7^3 \le N < 7^4$

(c) $\log_{10}(2.25) = \log_{10}(1.5)^2$

$= 2\log_{10}\left(\frac{3}{2}\right) = 2\,[\log_{10}3 - \log_{10}2)$

$= 2\,[0.4771 - 0.3010] = 2\,(0.1761)$

$= 0.3522$

(d) $\text{Antilog}_{2401} 0.75$

$= (2401)^{0.75} = (7^4)^{3/4} = 7^3 = 343$

Sol.5 **(a)** $y = 5^{200}$

$\Rightarrow \log_{10} y = 200\log_{10} 5 = 200\left[\log_{10} 5\times\frac{2}{2}\right]$

$= 200\,[\log_{10} 10 - \log_{10} 2] = 200\,[1 - 0.3010]$

$= 200\,[0.699] = 139.800$

$\Rightarrow \log_{10} y = 139.8$

y = number which have (139 + 1) = 140 positive integer

(b) $y = 6^{15} \Rightarrow \log_{10}y = 15[\log 3 + \log 2]$
$= 15[0.4771 + 0.3010] = 15 \times 0.7781$
$\Rightarrow \log_{10}y = 11.6715$
$\therefore$ y is 11 + 1 = 12 digit integer.

(c) $y = 3^{-100}$
$\log_{10}y == -100\log_{10}3$
$= -100 \times 0.4771 = -47.71$
$= -47 - 1 + 1 - 0.71 = \overline{48}\,.29$
y have no. of zeroes after the decimals is 48 – 1 = 47

Sol.6 $(\log_a x)(\log_a (xyz)) = 48$ ...(1)
$(\log_a y)\log_a(xyz) = 12,\ a > 0,\ a \neq 1$ ...(2)
$(\log_a z)\,.\,\log_a(xyz) = 84$ ...(3)
From (1) & (2), $\log_a x = 4\log_a y \Rightarrow x = y^4$
From (1) & (3), $\log_a z = 7\log_a y \Rightarrow z = y^7$
Put x & z in (2)
$\Rightarrow (\log_a y)(\log_a(y^4\,y\,y^7)) = 12$
$\Rightarrow (\log_a y)(\log_a y^{12}) = 12$
$\Rightarrow (\log_a y)(\log_a y) = 1 \Rightarrow \log_a y = \pm 1$

$\therefore\ y = a$ or $\frac{1}{a}$, $x = a^4$ or $\frac{1}{a^4}$, $z = a^7$ or $\frac{1}{a^7}$

Sol.7 $L = \text{antilog}_{1024}(0.4)$
$L = (1024)^{0.4} = (2^{10})^{0.4} = 2^4 = 16$
M = No. of digit in 6^{10}
$10[\log_{10}2 + \log_{10}3] = 10 \times 0.7781 = 7.781$
M = 8 (digit)
$\log_6 N = 2$ i.e. $\log_6 N < 3$
$6^2 \le N < 6^3$
No. of N is $= 6^3 - 6^2 = 6^2(6-1) = 36 \times 5 = 180$
L.M.N. = 16 × 8 × 180 = 23040

Sol.8 $\log_a x = x$ where $a = x^{\log_4 x}$
$x = a^x$ and $a^{\log_x 4} = x$
$\Rightarrow a^x = a^{\log_x 4} \Rightarrow x = \log_x 4 \Rightarrow x^x = 4$
$\Rightarrow x = 2$

Sol.9 $x^{\log_2 x + 4} = 32$
$\Rightarrow (\log_2 x + 4)\log_2 x = 5$
$\Rightarrow (\log_2 x)^2 + 4(\log_2 x) - 5 = 0$
$\Rightarrow [(\log_2 x) + 5][\log_2 x - 1] = 0$
$\Rightarrow \log_2 x = -5$ or $\log_2 x = 1$

$\therefore x = 2^{-5} = \frac{1}{32}$, $x = 2$

Sol.10 $\log_{x+1}(x^2 + x - 6)^2 = 4$
$\Rightarrow 2\log_{x+1}|x^2 + x - 6| = 4$
$\Rightarrow |x^2 + x - 6| = (x+1)^2$
$\Rightarrow \pm(x^2 + x - 6) = x^2 + 2x - 1$
If $x^2 + x - 6 = x^2 + 2x + 1 \Rightarrow x = -7$ not possible
If $-(x^2 + x - 6) = x^2 + 2x + 1$
$\Rightarrow 2x^2 + 3x - 5 = 0 \Rightarrow (2x+5)(x-1) = 0$

$\Rightarrow x = \frac{-5}{2}$ or $x = 1 \Rightarrow x = -\frac{5}{2}$ is reject so $x = 1$

Sol.11 $x + \log_{10}(1 + 2^x) = x\log_{10}5 + \log_{10}6$

$\Rightarrow \frac{5^x\,6}{1+2^x} = 10^x \Rightarrow 5^x\,.\,2^x\,(1 + 2^x) = 5^x\,.\,6$
$\Rightarrow 5^x[(2x)^2 + 2^x - 6] = 0$
$\Rightarrow 5^x \neq 0$ or $(2^x)^2 + 2^x - 6 = 0$
$2^x \neq -3$ or $2^x = 2$ so $x = 1$

Sol.12 $5^{\log_{10}x} - 3^{\log_{10}x-1} = 3^{\log_{10}x+1} - 5^{\log_{10}x-1}$

$\Rightarrow 5^{\log_{10}x} + \frac{5^{\log_{10}x}}{5} = 3^{\log_{10}x}\,.\,3 + \frac{3^{\log_{10}x}}{3}$

$\Rightarrow 5^{\log_{10}x}\left(\frac{6}{5}\right) = 3^{\log_{10}x}\left(\frac{10}{3}\right)$

$\Rightarrow \left(\frac{5}{3}\right)^{\log_{10}x} = \left(\frac{5}{3}\right)^2 \Rightarrow \log_{10}x = 2 \Rightarrow x = 100$

Sol.13 $\frac{1 + \log_2(x-4)}{\log_{\sqrt{2}}(\sqrt{x+3} - \sqrt{x-3})} = 1$

$\Rightarrow \log_2 2(x-4) = \log_{2^{1/2}}(\sqrt{x+3} - \sqrt{x-3})$

$\Rightarrow \log_2(2x-8) = \log_2(\sqrt{x+3} - \sqrt{x-3})^2$

$\Rightarrow 2x - 8 = (x-3) + (x-3) - 2\sqrt{x^2-9}$

$\Rightarrow \sqrt{x^2-9} = 4 \Rightarrow x = \pm 5$, (– 5 reject) so $x = 5$

Sol.14 $\log_5 120 + (x-3) - 2\log_5(1 - 5^{x-3})$
$= -\log_5(0.2 - 5^{x-4})$

$\Rightarrow \log_5 \frac{120(0.2 - 5^{x-4})}{(1 - 5^{x-3})^2} = (3 - x)$

$\Rightarrow \frac{120[(0.2 - 5^{x-4})]}{(1 - 5^{x-3})^2} = \frac{1}{5^{x-3}}$

$\Rightarrow \frac{120}{5}\,\frac{[1 - 5^{x-3}]}{[1 - 5^{x-3}]^2} = \frac{1}{5^{x-3}}$ Let $= 5^{x-3} = t$

$\Rightarrow \frac{24[1-t]}{[1-t]^2} = \frac{1}{t} \Rightarrow 24t = 1 - t \Rightarrow t = \frac{1}{25}$

$\Rightarrow\ 5^{x-3} = \frac{1}{25} = 5^{-2} \Rightarrow x - 3 = -2 \Rightarrow x = 1$

Sol.15 $\log 4 + \left(1+\frac{1}{2x}\right)\log 3 = \log(\sqrt[x]{3} + 27)$

$\Rightarrow\ 12t = t^2 + 27$ {Let $3^{\frac{1}{2x}} = t$}

$\Rightarrow\ t^2 - 12t + 27 = 0 \Rightarrow (t-9)(t-3) = 0$

$\therefore\ t = 9$ or $t = 3 \Rightarrow 3^{\frac{1}{2x}} = 3^2$ or $3^{\frac{1}{2x}} = 3^1$

$\Rightarrow \frac{1}{2x} = 2$ or $\frac{1}{2x} = 1 \Rightarrow x = \frac{1}{4}$ or $x = \frac{1}{2}$

$\Rightarrow x \notin N$ or $x \notin N \quad \therefore\ x \in \phi$

Sol.16 $2\log(2y-3x) = \log x + \log y$

$\Rightarrow (2y-3x)^2 = xy \Rightarrow 4y^2 + 9x^2 - 12xy = xy$

$\Rightarrow 9x^2 + 4y^2 - 13xy = 0 \Rightarrow (1)$

But $x > 0, y > 0$

$\Rightarrow 9\left(\frac{x}{y}\right)^2 + 4 - 13\left(\frac{x}{y}\right) = 0$

$\Rightarrow 9t^2 - 13t + 4 = 0$

$2y - 3x > 0$

$\frac{x}{y} < \frac{2}{3}$

$t = \frac{x}{y}$ (let)

$\Rightarrow (t-1)(9t-4) = 0 \Rightarrow \frac{x}{y} = 1, \frac{x}{y} = \frac{4}{9}$

$\frac{x}{y}$ should be $< \frac{2}{3}$; $\frac{x}{y} = 1$ (reject) $\Rightarrow \frac{x}{y} = \frac{4}{9}$

Sol.17 $\log_8 x + \log_4 y^2 = 5$ & $\log_8 y + \log_4 x^2 = 7$

$\Rightarrow \log_2 x^{1/3} + \log_2 y = 5$ & $\log_2 y^{1/3} + \log_2 x = 7$

$\Rightarrow x^{1/3}y = 2^5$ & $xy^{1/3} = 2^7$

multiply both equation

$\Rightarrow (xy)^{4/3} = 2^{12} \Rightarrow xy = 2^{12\times\frac{3}{4}} \Rightarrow xy = 2^9$

Sol.18 $\begin{cases}\log_{10}2 + \log x + \log y + \log x \log y = 4 - 3 \\ \log_{10}2 + \log y + \log z - \log y \log z = 1 \\ \log z + \log x - \log z \log x = 0\end{cases}$

{Let $\log_{10}x = X$, $\log_{10}y = Y$, $\log_{10}z = Z$}

$\Rightarrow \begin{cases}X + Y - XY = \log_{10}5 \\ Y + Z - YZ = \log_{10}5 \\ Z + X - XY = 0\end{cases}$

$\Rightarrow \begin{cases}(X-1)(Y-1) = 1 - \log_{10}5 = \log_{10}2 & \text{...(i)} \\ (Y-1)(Z-1) = 1 - \log_{10}5 = \log_{10}2 & \text{...(ii)} \\ (Z-1)(X-1) = 1 & \text{...(iii)}\end{cases}$

Muliply all equation (i), (ii), (iii)

$\Rightarrow [(X-1)(Y-1)(Z-1)]^2 = (\log_{10}2)^2$

$\Rightarrow (X-1)(Y-1)(Z-1) = \pm(\log_{10}2)$ ...(iv)

divided by (i), (ii) & (iii)

$\Rightarrow (Z-1) = \pm 1, (X-1) = \pm 1, (Y-1) = \pm \log_{10}2$

$Z = 0, 2$, $X = 0, 2$, $Y = 1 \pm \log_{10}2$

$\Rightarrow \log_{10}z = 0,\ \log_{10}x = 10$, $\log_{10}y = \log_{10}\frac{10}{2}$

$\Rightarrow z = 1, x = 1, y = 5$

and $\log_{10}z = 2, \log_{10}x = 2, \log_{10} = \log_{10}(10 \times 2)$

$\Rightarrow z = 100, x = 100, y = 20$

Sol.19 $x = 1 + \log_a bc$; $y = 1 + \log_b ca$, $z = 1 + \log_c ab$

$\Rightarrow x = \log_a abc$; $y = \log_b abc$; $z = \log_c abc$

$\Rightarrow x = \frac{1}{\log_{abc} a}$ $y = \frac{1}{\log_{abc} b}$; $z = \frac{1}{\log_{abc} c}$

we wish to prove that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

L.H.S. $= \log_{abc} abc = 1$

Sol.20 Let $\log_{(c+b)} a + \log_{(c-b)} a = 2\log_{c+b} a \log_{c-b} a$

$\Rightarrow \log_a(c-b) + \log_a(c+b) = 2$

$\Rightarrow c^2 - b^2 = a^2 \Rightarrow a^2 + b^2 = c^2$

Sol.21 $\frac{\log_N c}{\log_N a} = \frac{\frac{1}{\log_N a} - \frac{1}{\log_N b}}{\frac{1}{\log_N b} - \frac{1}{\log_N c}}$

$\Rightarrow \frac{\log_N c}{\log_N a} = \frac{\log_N b . \log_N c}{\log_N a . \log_N b}\frac{\left(\log_N \frac{b}{a}\right)}{\left(\log_N \frac{c}{b}\right)}$

$\Rightarrow \log_N \frac{b}{a} = \log_N \frac{c}{b} \Rightarrow \frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac$

Sol.22 $\frac{3}{2}\log_4(x+2)^2 + 3 = \log_4(4-x)^3 + \log_4(6+x)^3$

$\Rightarrow 3\log_4|x+2| + 3 = 3\log_4(4-x) + 3\log_4(6+x)$

$\Rightarrow \log_4|x+2| + \log_4 4 = \log_4(4-x)(6+x)$

$\Rightarrow 4|x+2| = (4-x)(6+x)$

(i) If $x + 2 \geq 0$ then $4(x+2) = 24 - 2x - x^2$

$\Rightarrow x^2 + 6x - 16 = 0 \Rightarrow (x+8)(x-2) = 0$

$\therefore\ x = -8, 2 \because (-8+2) \not\geq 0$ so $x = 2$

(ii) If $x + 2 < 0$ then $-4(x + 2) = (4 - x)(6 + x)$

$\Rightarrow -4x - 8 = 24 - 2x - x^2 \Rightarrow x^2 - 2x - 32 = 0$

$\Rightarrow x = \dfrac{2 \pm \sqrt{4+128}}{2} = 1 \pm \sqrt{33}$

$\because (1 + \sqrt{33}) + 2 \not< 0$ so $x = 1 - \sqrt{33}$

Sol.23 $\sqrt{(2008)}\,(x)^{\log_{2008} x} = x^2$ (here $x > 0$)

taking $\log_{2008}$ both side

$\Rightarrow \dfrac{1}{2} + (\log_{2008}x)^2 = 2(\log_{2008}x)$

$\Rightarrow 2(\log_{2008}x)^2 - 4(\log_{2008}x) + 1 = 0$

product of roots $\alpha\beta$

$\log_{2008}\alpha\beta = (\log_{2008}\alpha) + (\log_{2008}\beta)$

$\Rightarrow \log_{2008}\alpha\beta = -\left(-\dfrac{4}{2}\right) \quad \therefore \alpha\beta = (2008)^2$

Sol.24 $\log^2\left(\dfrac{x+4}{x}\right) + \log^2\left(\dfrac{x}{x+4}\right) = 2\log^2\left(\dfrac{3-x}{x-1}\right)$

$\{\log^2 \dfrac{a}{b} = \log^2 \dfrac{b}{a}\}$

$2\log^2\left(\dfrac{x+4}{x}\right) = 2\log^2\left(\dfrac{3-x}{x-1}\right)$

$\left[\log\left(\dfrac{x+4}{x}\right) + \log\left(\dfrac{3-x}{x-1}\right)\right]\left[\log\left(\dfrac{x+4}{x}\right) - \log\left(\dfrac{3-x}{x-1}\right)\right] = 0$

$\log \dfrac{(x+4)(3-x)}{x(x-1)} \,.\, \log \dfrac{(x+4)(x-1)}{x(3-x)} = 0$

$\dfrac{(x+4)(3-x)}{x(x-1)} = 1$ or $\dfrac{(x+4)(x-1)}{x(3-x)} = 1$

$-x^2 - x + 12 = x^2 - x$ or $x^2 + 3x - 4 = -x^2 + 3x$

$2x^2 = 12 \Rightarrow x = \pm\sqrt{6}$ or $2x^2 = 4 \Rightarrow x = \pm\sqrt{2}$

$\because \left(1 + \dfrac{4}{x}\right) > 0$ So $x = -\sqrt{6}$; $-\sqrt{2}$ reject

$\therefore x = \sqrt{2}, \sqrt{6}$

Sol.25 $\log_3(\sqrt{x} + |\sqrt{x} - 1|) = \log_9(4\sqrt{x} - 3 + 4|\sqrt{x} - 1|)$

$\Rightarrow \log_9(\sqrt{x} + |\sqrt{x} - 1|)^2$

$= \log_9(4\sqrt{x} - 3 + 4|\sqrt{x} - 1|)$

$\Rightarrow (\sqrt{x} + |\sqrt{x} - 1|)^2 = 4\sqrt{x} - 3 + 4|\sqrt{x} - 1|$

$\Rightarrow x + (\sqrt{x} - 1)^2 + 2\sqrt{x}\,|\sqrt{x} - 1|$

$= 4\sqrt{x} + 4|\sqrt{x} - 1| - 3$

$\Rightarrow x + x + 1 - 2\sqrt{x} + 2\sqrt{x}\,|\sqrt{x} - 1|$

$= 4\sqrt{x} + 4|\sqrt{x} - 1| - 3$

$\Rightarrow 2x - 6\sqrt{x} + 2\sqrt{x}\,|\sqrt{x} - 1| = 4|\sqrt{x} - 1| - 3 - 1$

$\Rightarrow x - 3\sqrt{x} + 2 = |\sqrt{x} - 1|\,(2 - \sqrt{x})$

$\Rightarrow (\sqrt{x} - 1)(\sqrt{x} - 2) + |\sqrt{x} - 1|(\sqrt{x} - 2) = 0$

If $\sqrt{x} - 1 \ge 0 \Rightarrow x \ge 1$

$2(\sqrt{x} - 1)(\sqrt{x} - 2) = 0 \Rightarrow \sqrt{x} = 1$ or $\sqrt{x} = 2$

$\Rightarrow x = 1$ or $x = 4$

If $\sqrt{x} - 1 < 0 \Rightarrow x < 1$

$(\sqrt{x} - 1)(\sqrt{x} - 2) - (\sqrt{x} - 1)(\sqrt{x} - 2) = 0 \Rightarrow 0 = 0$

identity for $x < 1$

but $\sqrt{x}$ include in equation so $x \ge 0$

common interval of $x \ge 0$ and $x < 1$

$0 \le x < 1$ & $x = 1$, $x = 4$ so $[0, 1], \{4\}$

Sol.26 $\log^2(4 - x) + \log(4 - x)\log\left(x + \dfrac{1}{2}\right) - 2\log^2\left(x + \dfrac{1}{2}\right) = 0$

Let $A = (4 - x)$ & $B = \log\left(x + \dfrac{1}{2}\right)$

$\Rightarrow A^2 + AB - 2B^2 = 0 \Rightarrow (A - B)(A + 2B) = 0$

so $\log(4 - x) = \log\left(x + \dfrac{1}{2}\right) \Rightarrow 4 - x = x + \dfrac{1}{2}$

$\Rightarrow 2x = \dfrac{7}{2} \qquad \Rightarrow x = \dfrac{7}{4}$

or $\log(4 - x) + 2\log\left(x + \dfrac{1}{2}\right) = 0$

$\Rightarrow (4 - x)(x + \dfrac{1}{2})^2 = 1 \Rightarrow -4x^3 + 12x^2 + 15x = 0$

$\Rightarrow x = 0$; $4x^2 - 12x - 15 = 0$

$x = \dfrac{12 \pm \sqrt{144 + 240}}{8} = \dfrac{12 \pm 4\sqrt{24}}{8}$

$x = \dfrac{3 \pm \sqrt{24}}{2}$ $\left[\because\ x + \dfrac{1}{2} > 0 \text{ so } \dfrac{3-\sqrt{24}}{2} \text{ is reject}\right.$

Sol.27 Let a number M such that

$\log_{10} M \geq p \quad \Rightarrow \quad M \geq 10^p$

$\log_{10} M < p + 1 \Rightarrow \quad M < 10^{p+1}$

P = No. of integer M is = $10^{p+1} - 10^p$

$P = 10^p . 9$

Let a number N such that

$\log_{10}\left(\dfrac{1}{N}\right) > -q$ & $\log_{10}\left(\dfrac{1}{N}\right) < -q + 1$

$\Rightarrow \dfrac{1}{N} > 10^{-q}$ & $\dfrac{1}{N} < 10^{-q+1} \Rightarrow N < 10^q$ & $N > 10^{q-1}$

Q = Number of integer N is = $10^q - 10^{q-1}$

$= 10^{q-1}(10 - 1) = 10^{q-1} . 9$

$\log_{10}P - \log_{10}Q = \log_{10}\dfrac{P}{Q} = \log_{10}\dfrac{10^p.9}{10^{q-1}.9}$

$= \log_{10}10^{p-q+1} = p - q + 1$

EXERCISE – V HINTS & SOLUTIONS

Sol.1 $\log_{3/4}\log_8(x^2+7) + \log_{1/2}\log_{1/4}(x^2+7)^{-1} = -2$

$\Rightarrow \log_{3/4}\left(\dfrac{1}{3}\log_2(x^2+7)\right) + \log_{1/2}\left(\dfrac{1}{2}\log_2(x^2+7)\right) = -2$

$\Rightarrow \log_{3/4}\left(\dfrac{t}{3}\right) + \log_{1/2}\left(\dfrac{t}{2}\right) = -2$ {Let $\log_2(x^2+7) = t$}

$\Rightarrow \dfrac{\log_2 t - \log_2 3}{\log_2 3 - 2\log_2 2} + \dfrac{\log_2 t - 1}{\log_2\left(\dfrac{1}{2}\right)} = -2$

$\Rightarrow \dfrac{z-a}{a-2} + \dfrac{z-1}{-1} = -2$ {Let $\log_2 t = z$ & $\log_2 3 = a$}

$\Rightarrow z - a + (1 - z)(a - 2) = -2(a - 2)$

$\Rightarrow z(3 - a) - a + a - 2 = -2a + 4$

$\Rightarrow z(3 - a) = -2a + 6 \Rightarrow z(3 - a) = 2(3 - a)$

$\Rightarrow z = 2 \Rightarrow \log_2 t = 2 \Rightarrow t = 4$

& $\log_2(x^2 + 7) = 4$

$\Rightarrow x^2 + 7 = 2^4 \Rightarrow x^2 = 9 \Rightarrow x = -3, 3$

Sol.2 $\log_4(x - 1) = \log_2(x - 3)$

$\Rightarrow \dfrac{1}{2}\log_2(x-1) = \log_2(x-3)$

$\Rightarrow \log_2(x - 1) = \log_2(x - 3)^2$

$\Rightarrow x - 1 = x^2 - 6x + 9 \Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow (x - 2)(x - 5) = 0 \Rightarrow x = 2, 5$ but $x > 3$

$\Rightarrow x = 5$ only one solution.

Answer Ex–I SINGLE CORRECT (OBJECTIVE QUESTIONS)

1. C **2.** A **3.** D **4.** B **5.** A **6.** B **7.** D
8. C **9.** B **10.** A **11.** C **12.** D **13.** B **14.** C
15. D **16.** D **17.** D **18.** B **19.** C **20.** B **21.** C
22. C

Answer Ex–II MULTIPLE CORRECT (OBJECTIVE QUESTIONS)

1. AB **2.** ABCD **3.** AB **4.** ABCD **5.** ABCD

Answer Ex–III SUBJECTIVE QUESTIONS

2. **(i)** $\sqrt{2}-1$ **(ii)** $\frac{2+\sqrt{2}-\sqrt{6}}{4}$

3. **(i)** $(x-2y)(x^2+y^2-xy)$ **(ii)** $\left(a-\frac{1}{a}+1\right)\left(a^2+\frac{1}{a^2}-a+\frac{1}{a}+2\right)$ **(iii)** $(x-1)(x-2)(x-3)$

(iv) $(x+2)(x^2-2x-5)$ **(v)** $-(a-b)(b-c)(c-a)$

4. **(i)** $(x^4-x^2+1)(x^2+x+1)(x^2-x+1)$ **(ii)** $(x^2-2x+2)(x^2+2x+2)$ **5.** $\frac{a^4}{b^4}$

6. $a_1 = a_2 = a_3 = \ldots\ldots\ldots\ldots = a_n = 0$ **7.** **(i)** $x = \pm 1$ **(ii)** $x = 5$ **(iii)** $x = 2$ **(iv)** $x = -3, 3$ **(v)** $x = -1$

8. 0 **9.** $x = 10$ **10.** $a\,b\,c = 1$ **11.** 1 **13.** **(a)** $\log_2 3$ **(b)** $\log_7 11$

14. $x = 16$ or $x = -4$ **15.** 8 **16.** {1/3} **17.** {–4}

18. $\frac{1}{20}, \frac{1}{5}$

19. $\{10^{-5} .10^3\}$ **20.** $\left\{1, 5, \frac{1}{25}\right\}$ **21.** $x = 16$ **22.** $x = 3$ **23.** **(a)** 12 **(b)** 47

24. $x = 10/3,\ y = 20/3$ & $x = -10,\ y = 20$ **25.** $x = 2$ or 81 **26.** $x = 8$ **27.** 12

28. **(a)** $\frac{xy+2}{2y}, \frac{xy+2y-2}{2y}$ **(b)** 625 **29.** **(a)** $x = 16$ or $x = -4$ **(b)** $x = 5$

30. **(a)** –1 **(b)** $\log_b N$ **31.** **(a)** 8 **(b)** $x = 3$ **32.** **(a)** 2 **33.** 2

34. 3721 **35.** $\frac{1}{6}$ **36.** 9 **37.** 1 **38.** 6

39. $x = \frac{1}{100}$

40. $2s + 10s^2 - 3(s^3 + 1)$ **41.** $\frac{25}{2}$ **42.** $\frac{1+2ac}{2c+abc+1}$

Answer Ex–IV ADVANCED SUBJECTIVE QUESTIONS

2. **(a)** x = 5 **(b)** x = 10 **(c)** x = $2^{\sqrt{2}}$ or $2^{-\sqrt{2}}$ **(d)** x = $2^{-\log a}$ where base of log is 5.

3. 507 **4.** **(a)** 0.5386l $\bar{1}.5386$; $\bar{3}.5386$ **(b)** 2058 **(c)** 0.3522 **(d)** 343

5. **(a)** 140 **(b)** 12 **(c)** 47 **6.** (a^4, a, a^7) or $\left(\frac{1}{a^4}, \frac{1}{a}, \frac{1}{a^7}\right)$ **7.** 23040

8. x = 2 **9.** x = 2 or $\frac{1}{32}$ **10.** x = 1 **11.** x = 1 **12.** x = 100

13. x = 5 **14.** x = 1 **15.** $x \in \phi$ **16.** 4/9 **17.** xy = 2^9

18. x = 1, y = 5, z = 1 or x = 100, y = 20, z = 10 **22.** x = 2 or 1 – $\sqrt{33}$

23. $(2008)^2$ **24.** x = $\sqrt{2}$ or $\sqrt{6}$ **25.** $[0, 1] \cup \{4\}$ **26.** $\left\{0, \frac{7}{4}, \frac{3+\sqrt{24}}{2}\right\}$ **27.** p – q + 1

Answer Ex–V JEE PROBLEMS

1. x = 3 or – 3 **2.** B